Effective Widths and Effective Number of Phonons of Multiphonon Giant Resonances*

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Abstract

We discuss the origin of the difference between the harmonic value of the width of the multiphonon giant resonances and the smaller observed value. Analytical expressions are derived for both the effective width and the average cross-section. The contribution of the Brink-Axel mechanism in resolving the discrepancy is pointed out.

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In a series of publications [1–3], we have advanced the idea that the excitation of the double giant dipole resonances (DGDR) in heavy-ion collisions proceeds in two distinct, incoherent, ways: the usual sequential excitation via the single giant dipole resonance (GDR), usually referred to in the literature as the "harmonic" cross section [4], and the fluctuation excitation that involves the "internal" decay of the GDR into the complicated background states followed by a collective excitation of a GDR on these states, a manifestation of the Brink-Axel mechanism. Of course in a fully microscopic approach to the excitation process, the Brink-Axel effects are also contained. However, the usefulness of the approach of [1,3] is the clear identification of the average cross-section with a "direct" excitation and the fluctuation one with the higher-order "non-direct" contribution. This allows identifying different time scales with the excitation process which makes possible an unambiguous treatment of the eventual decay of the multiphonon states [2]. The question of the "enhancement" of the experimental cross-section with respect to the harmonic value is easily addressed within the average+fluctuation model [1,3]. However, the enhancement problem, especially in ^{136}Xe still remains not fully resolved. There has also been some discussion in recent years related to the fact that the observed width of the double giant dipole resonance deviates from the harmonic value of twice the single giant resonance width. As we have commented previously, [1] this is a natural result of the incoherent contributions to the cross section, even when the resonances themselves are purely harmonic. In fact, due to the energy dependence of the incoherent contributions to the excitation cross sections, we expect the effective n-phonon width to be energy dependent. The purpose of this paper is to derive an analytical expression for the effective energy-dependent width of the n-phonon excitation distribution. For this purpose we use multistep distorted wave series in conjunction with statistical averaging that allows us to significantly extend the result of our previous publication [1]. The method enables us to calculate the inclusive n-phonon cross-section in closed form and allows us to identify the physical quantity responsible for the cross-section enhancement. This quantity is the product of the damping width of the single phonon resonance times the collision time evaluated at the grazing impact parameter b_o.

It is convenient to introduce projection operators: P will stand for the entrance channel, d_1 the 1 phonon channel, q_1 the corresponding fine structure subspace etc. We use multistep

distorted wave picture. Thus for the excitation of e.g. 2 phonons, one has

$$\sigma^{(2)} = \left| \left\langle x_f^{(-)}, f \left| V \; G_1^{(+)} \; V \right| x_i^{(+)}, 0 \right\rangle \right|^2 \; . \tag{1}$$

In the above V stands for the Coulomb interaction responsible for the excitation, $x^{(+)}$ stands for the appropriate distorted wave and $G_1^{(+)}$ is the Green's operator in the subspace $d_1 + q_1$. This is given by

$$G_1^+ = \begin{pmatrix} g^{(+)}_{d_1 d_1} & G^{(+)}_{d_1} \; v_{d_1 q_1} \; g^{(+)}_{q_1 q_1} \\ G^{(+)}_{q_1} \; v_{q_1 d_1} \; g^{(+)}_{d_1 d_1} & g^{(+)}_{q_1 q_1} \end{pmatrix}, \tag{2}$$

where, $g^{(+)}_{ii} = \left(E^{(+)} - H_i - v_{ij} G^+_j v_{ji}\right)^{-1}$, $G^{(+)}_i = \left(E^{(+)} - H_i\right)^{-1}$, $H_i = h_{rel} + h_i$, and $E^{(+)} = E + i\varepsilon$, with h_{rel} representing the Hamiltonian of relative motion and h_i the nuclear Hamiltonian of ion i. The interaction v, couples the different subspaces.

The interaction V connects the ground state to the one-phonon doorway state d_1. It can also connects a state in $q_1$to a one-phonon Brink-Axel state. Thus the amplitude that enters into the definition of the cross-section $\sigma^{(2)}$ separates into two pieces

$$\begin{aligned} \sigma^{(2)} = & \left| \left\langle x_f^{(-)}, 2 | V_{d_2 d_1} \left(E^{(+)} - H_{d_1} - v_{d_1 q_1} G_{q_1} v_{q_1 d_1}\right)^{-1} V_{d_1,0} \right| x_i^{(+)}, 0 \right\rangle \\ & + \left\langle x_f^{(-)}, 1 \left| V_{d_1 q_1} \frac{1}{E^{(+)} - H_{q_1}} v_{q_1 d_1} \frac{1}{E^{(+)} - H_{d_1} - v_{d_1 q_1} G_{q_1} v_{q_1 d_1}} V_{d_1,0} | x_i^{(+)}, 0 \right\rangle \right|^2 \; . \end{aligned} \tag{3}$$

The fine structure states $\{q_1\}$ are quite complicated many particle-many hole configurations. In this respect, a statistical treatment of the second, fluctuation, contribution is called for. Thus we take the energy average of $\sigma^{(2)}$ to represent the theoretical cross-section to be compared to the data. When performing the energy average, the cross term in Eq. (4) vanishes since it is linear in the assumed random coupling $v_{q_1 d_1}$. Thus $\overline{\sigma^{(2)}}$ is given, as usual, by the incoherent sum of two terms. The first term is the coherent "harmonic" cross-section. Here the Green's function $(E - H_{d_1} - v_{d_1 q_1} G_{q_1} v_{q_1 d_1})^{-1}$ is replaced by an average over the q_1 states $\left(E - H_{d_1} - \overline{v_{d_1 q_1} G_{q_1} v_{q_1 d_1}}\right)^{-1}$. The correction to this approximation involves fluctuations that corresponds to the process $d_1 \rightarrow q_1 \rightarrow d_1$, which is negligible. The average $\overline{v_{d_1 q_1} G_{q_1} v_{q_1 d_1}}$ is as usual written as $\Delta_{d_1} - i\frac{\Gamma_{d_1}}{2}$, which defines the resonance shift Δ_{d_1} and width Γ_{d_1} respectively. These two quantities satisfy a dispersion relation. The fluctuation contribution to $\overline{\sigma^{(2)}}$ involves an average over the q_1 states.

This average is easily performed if we recall that the q_1 Green's function is a large sum of random contributions. Thus, writing first

$$G_q^{(+)} = \sum_{q_1} \frac{|q_1\rangle \langle q_1|}{E^{(+)} - \varepsilon_{q_1} + i\frac{\Gamma_{0q_1}}{2} - h_{rel}} \equiv \int \frac{d\vec{k}}{(2\pi)^3} \sum_{q_1} \frac{\left|x_k^{(+)}\right\rangle |q_1\rangle \langle q_1| \left\langle x_k^{(+)}\right|}{E^{(+)} - \varepsilon_{q_1} + i\frac{\Gamma_{0q_1}}{2} - E_k} \ , \tag{4}$$

where we have employed the scattering eigenstates of h_{rel}, $\left\{\left|x_k^{(+)}\right\rangle\right\}$, we can reduce the average above into

$$\overline{\sigma_{fl}^{(2)}} = \int \frac{d\vec{k}}{(2\pi)} \int \frac{d\vec{k'}}{(2\pi)^3} \left\langle 0, x_i^{(+)} \left| V^\dagger g_{d_1}^\dagger \right| d_1 \right\rangle \overline{\sum_{q_1} \frac{\left\langle d\left|v^\dagger\right| q_1 x_k^{(+)}\right\rangle \left\langle q_1 x_k^{(+)}\left|V^\dagger\right| x_f^{(-)}, 1\right\rangle}{E - \varepsilon_{q_1} - i\frac{\Gamma_{0q_1}}{2} - E_k}} \cdot \overline{\sum_{q_1'} \frac{\left\langle x_f^{(-)}, 1\left|V\right| q\prime, x_{k\prime}^{(+)}\right\rangle \left\langle q\prime, x_{k\prime}^{(+)}\left|v\right| d_1\right\rangle}{E - \varepsilon_{q_1'} + i\frac{\Gamma_{0q_1'}}{2} - E_{k\prime}}} \left\langle d_1 \left|g_d V\right| 0, x_i^{(+)}\right\rangle \ . \tag{5}$$

The average over $\{q_1\}$ makes the double q_1 sum collapses into a single sum ($q_1 = q_1'$). Further, since there are many q_1 states, we replace the q_1 sum by an integral $\sum_{q_1} = \int d\varepsilon_{q_1} \rho\left(\varepsilon_{q_1}\right)$, where $\rho\left(\varepsilon_{q_1}\right)$ is the density of the q_1 states. The ε_q integral can be easily performed if we assume that the numerator is slowly varying. Only the two $\varepsilon_q - poles$ will contribute. Thus

$$\overline{\sigma_{fl}^{(2)}} = \Gamma_{d_1}^{\downarrow} \int \frac{d\vec{k}}{(2\pi)^3} \int \frac{d\vec{k'}}{(2\pi)^3} \left\langle 0 x_i^{(+)} \left| V^\dagger g_{d_1}^\dagger \right| d_1 x_k^{(+)} \right\rangle \left\langle \tilde{0}\left(\overline{\varepsilon}_q\right) x_k^{(+)} \left|V^\dagger\right| 1, x_f^{(-)} \right\rangle \cdot \frac{1}{\left|-\overline{\Gamma}_{0q_1} + i\left(E_k - E_{k'}\right)\right|} \left\langle x_f^{(-)}, 1\left|V\right| \tilde{0}\left(\overline{\varepsilon}_q\right) x_{k'}^{(+)}\right\rangle \left\langle x_{k'}^{(+)} d_1 \left|g_{d_1} V\right| 0 x_i^{(+)}\right\rangle \ , \tag{6}$$

where we have introduced the spreading width of the doorway state d_1, defined by

$$\Gamma_{d_1}^{\downarrow} = 2\pi \rho_{q_1} \overline{|\langle q_1| v |d_1\rangle|^2} \ , \tag{7}$$

and also introduced a representative excited intrinsic state $\left|\tilde{0}\left(\overline{\varepsilon}_q\right)\right\rangle$, which acts as a ground state for the Brink-Axel phonon excitation.

The energy correlation function $\left|-\Gamma_{0q_1} + i\left(\varepsilon_k - \varepsilon_{k'}\right)\right|^{-1}$ controls the magnitude of $\sigma_{fl}^{(2)}$. For $\varepsilon_k = \varepsilon_{k'}$, and $\Gamma_{0q_1} \to 0$, the contribution is much smaller than $\sigma_h^{(2)}$ since only one $\vec{k}$ integral survive in the former. On the other hand, if $\overline{\Gamma}_{0q_1}$ is much larger than the range of values of $\varepsilon_k - \varepsilon_{k'}$ which an relevant for the integral, then $\overline{\sigma}_{fl}^{(2)}$ would be $\frac{\Gamma_{d_1}^{\downarrow}}{\overline{\Gamma}_{0q_1}}$ times a regular

cross section. However, $\overline{\Gamma}_{0q_1}$ is by definition $\ll \Gamma_d^{\downarrow}$, and therefore this last case does not occur. Therefore we may simply say that the correlation function introduces a characteristic time, the "correlation" time, τ_c which depends on the bombarding energy. Accordingly we write for $\overline{\sigma_{fl}^{(2)}}$ the following

$$\overline{\sigma_{fl}^{(2)}} = \Gamma_{d_1}^{\downarrow} \tau_c (E) \left| \left\langle x_f^{(-)}, 1 \left| V \right| \tilde{0} (\varepsilon_{d_1}) \right\rangle \left\langle d_1 | g_d V | 0 x_i^{(+)} \right\rangle \right|^2 , \tag{8}$$

where the integrals $\int \frac{d\vec{k}'}{(2\pi)^3} \int \frac{d\vec{k}}{(2\pi)^3} \left| x_{k'}^{(+)} \right\rangle \left| x_k^{(+)} \right\rangle \left\langle x_k^{(+)} \right| \left\langle x_{k'}^{(+)} \right|$ have been set equal to unity since the distorted waves sts $\{x^{(+)}\}$ is complete.

Now since $\left\langle x_f^{(-)}, 1 \left| V \right| \tilde{0} (\varepsilon_{d_1}) \right\rangle$, contains one Brink-Axel phonons in the final state, where as the corresponding amplitude in the harmonic cross-section, $\left\langle x_f^{(-)}, 2 \left| V \right| 1 \right\rangle$, contains 2, we can make the approximation $\frac{\left\langle x_f^{(-)}, 1 | V | \tilde{O} \right\rangle}{\left\langle x_f^{(-)}, 2 | V | 1 \right\rangle} \cong \frac{1}{\sqrt{2!}}$.

Thus we find for $\overline{\sigma_{fl}^{(2)}}$, the following reasonable approximate form

$$\overline{\sigma_{fl}^{(2)}} = \frac{1}{2} \frac{\Gamma_{d_1}^{\downarrow} \tau_c (E)}{\hbar} \sigma_c^{(2)} . \tag{9}$$

Thus

$$\overline{\sigma^{(2)}} = \sigma_c^{(2)} \left(1 + \frac{1}{2} \frac{\Gamma_{d_1}^{\downarrow} \tau_c (E)}{\hbar} \right) . \tag{10}$$

The second term above gradually becomes insignificant as the bombarding energy increases.

The calculation of the cross-section for higher number of phonons follows a similar procedure. Keeping track of the number of routes that can be followed to reach the final "state", we have for $\sigma^{(3)}$

$$\overline{\sigma^{(3)}} = \sigma_c^{(3)} \left(1 + \frac{2}{3} \frac{\Gamma_{d_1}^{\downarrow} \tau_c (E)}{\hbar} + \frac{1}{3} \left(\frac{\Gamma_{d_1}^{\downarrow} \tau_c (E)}{\hbar} \right)^2 \right) . \tag{11}$$

The generalization to n-phonon is straightforward

$$\begin{aligned} \overline{\sigma^n} &= \sigma_h^{(n)} + \sigma_c^{(n)} \sum_{k=1}^{n} \frac{(n-k)!}{n!} \frac{n-k}{n+k} \binom{n}{k} \left(\frac{\Gamma_{d_1} \tau_c (E)}{\hbar} \right)^k \\ &\equiv \sigma_c^{(n)} + \sum_{k=1}^{n} \sigma_{fl}^{(n)} (k) . \end{aligned} \tag{12}$$

Note that $\sigma_c^{(n)}$ is the cross-section for the excitation of n-phonon states that proceeds though $n-1, n-2, ...,$ phonon states which have finite life times (width). In this respect we are generalizing the concept of harmonic cross-section. The consideration of the width of the intermediate states and the fluctuations that result from the decay of these states go hand in hand.

Our analytical formula for the average cross-section is quite reasonable when compared to numerical solution of the evolution equation for the density matrix of the system reported in ref. [3] if the correlation time is identified with the collision time at the grazing impact parameter, $\tau_c = b_o/\gamma v$, Fig. 1. In view of this we are confident in applying our theory to calculate other observables such as the width of the final channel.

Taking a Breit-Wigner form for the spectrum of each of the component in (12) and evaluating this at the peak, we can define an effective width as

$$\frac{\sigma_c^{(n)}}{n\Gamma_{d_1}} + \sum_{k=1}^{n-1} \frac{\sigma_{fl}^{(n)}(k)}{(n-k)\,\Gamma_{d_1}} = \frac{\sigma^{(n)}}{\Gamma_{eff}^{(n)}}\ , \tag{13}$$

or

$$\Gamma_{eff}^{(n)}(E) = \left(\frac{\sigma_h^{(n)} + \Sigma_k \sigma_{fl}^{(n)}(k)}{\sigma_c^{(n)} + \sum\limits_{k=1}^{n-1} \frac{n}{n-k} \sigma_{fl}^{(n)}(k)} \right) n\Gamma_{d_1} = \frac{1}{1 + \frac{\sum\limits_{k=1}^{n-1} \frac{k}{n-k}\, \sigma_{fl}^{(n)}(k)}{\sigma^{(n)}}} \left(n\Gamma_{d_1}\right)\ . \tag{14}$$

Eq. (14) for $\Gamma_{eff}^{(n)}(E)$ shows that the effective width attains the harmonic value $n\Gamma_{d_1}$ at high energies. At low bombarding energy the value can be significantly smaller than $n\Gamma_{d_1}$, owing to the Brink-Axel reduction factor $\left(1 + \frac{\sum\limits_{k=1}^{n} \frac{k}{n-k}\, \sigma_{fl}^{(n)}(k)}{\sigma^{(n)}}\right)^{-1}$. This is demonstrated in Fig. 2. The B-A mechanism thus supplies a correlation between the experimental effective width and the corresponding cross-section. This is an important result which further helps shed light on the excitation mechanism. For two phonons we have $\frac{\sigma^{(2)}}{\sigma_c^{(2)}} = \frac{1+\frac{\Gamma_{eff}}{2\Gamma_1}}{2\left(\frac{\Gamma_{eff}}{2\Gamma_1}\right)}$. This relation is independent of the system. In figure (3) we exhibit the available data taken from ref. [6]. In presenting the data, we have changed the variable $\frac{\sigma^{(2)}_{\exp.}}{\sigma^{(2)}_{harm.}}$ into $\frac{\sigma^{(2)}_{\exp.}}{\sigma_c^{(2)}}$, with $\sigma^{(2)}_{harm.}$being the value of $\sigma_c^{(2)}$ when the width of the intermediate one-phonon state is set equal to zero. Although it is claimed that $\Gamma^{(2)}_{eff} = \sqrt{2}\Gamma_1$, our result does not exclude a more subtle connection between $\Gamma^{(2)}_{eff}$ and Γ_1.

The above discussion suggests defining an effective phonon number $n_{eff}^{(E)} \equiv \frac{n}{\frac{1+\sum_{k=1}^{n-1}\frac{k}{n-k}\sigma_{fl}^{(n)}(k)}{\sigma^{(n)}}}$ which is smaller than n. This gives an immediate qualitative explanation for the enhancement in the cross-section too. Since $n_{eff} < n$, the energy $E_{n_{eff}} < nE_1$. Accordingly, if one were to use damped anharmonic oscillator model with the following sequence of energies, $0 \rightarrow E_1 \rightarrow 2E_1 - \frac{\Gamma\tau}{1+\Gamma\tau}E_1 \rightarrow 3E_1 - \frac{\Gamma\tau+2(\Gamma\tau)^2}{1+\Gamma\tau+(\Gamma\tau)^2}E_1$, ..., one would obtain larger cross-sections $\sigma^{(2)}$; $\sigma^{(3)}$, ... when compared to $\sigma_c^{(2)}$, $\sigma_c^{(3)}$.... In a way, this supplies a simple connection between the direct + fluctuation model and the anharmonic models [7-9]. In Fig. 4 we show the result of a calculation following this line. The multiphonon cross-section calculated for the effective damped anharmonic oscillator (long-dashed line), whose spectrum is given by $E_{n_{eff}} = n_{eff}\ E_1$. The semiclassical coupled channels model [4b] is used to generate this result. The widths of the intermediate states are just the "harmonic" ones $\Gamma_n = n\Gamma_1$. Also shown is the cross-section from Ref. [3] (solid line) and the simple undamped harmonic oscillator cross-section also calculated according to [4b] (small dashed line). The results shown are for the system ^{208}Pb+^{208}Pb. The agreement of our damped anharmonic oscillator, coupled channels calculation with those of Ref. [3], which is based on the solution of an evolution equation for the excitation probability with loss and gain terms, is excellent.

In conclusion, the direct+fluctuation model (Brink-Axel mechanism) supplies a natural framework to discuss the cross-section enhancement and the width reduction of the multiphonon states. The enhancement factor of the cross-section $\frac{\sigma^{(n)}}{\sigma_c^{(n)}}$ and the width reduction $\frac{\Gamma_{eff}^{(n)}}{n\Gamma_{d_1}}$ can be co-related by the effective number of phonons. This concept should be quite useful in future work on multiphonon physics. Further, our results here should also be relevant to other multistep reaction phenomena such as preequilibrium processes [10].

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FIGURE CAPTIONS

Fig. 1: The quantity $\Gamma^{\downarrow}\tau_c$ as a function of the bombarding energy for the ^{208}Pb + ^{208}Pb system.

Fig. 2: The effective observed width $\Gamma^{(n)}_{eff}$, Eq. (14) vs. the bombarding energy for the ^{208}Pb+^{208}Pb system. Clearly $\Gamma^{(1)}_{eff} \equiv \Gamma\left(d_1\right)$, and it is set equal to 4.0MeV. See text for details.

Fig. 3: The effective width reduction vs. the cross-section enhancement. The data are from Ref. [6]. The "data" for the cross-section ratios is $\frac{\sigma^{(2)}_{\exp.}}{\sigma^{(2)}_{c}}$ which is just the values of Ref. [6], $\frac{\sigma^{(2)}_{\exp.}}{\sigma^{(2)}_{harm.}}$, multiplied by $\frac{\sigma^{(2)}_{c}\left(\Gamma_{d_1}=0\right)}{\sigma^{(2)}_{c}}$, since $\sigma^{(2)}_{harm.} \equiv \sigma^{(2)}_{c}\left(\Gamma_{d_1}=0\right)$. See text for details.

Fig. 4: The multiphonon cross-section calculated for the effective damped anharmonic oscillator (long-dashed line), whose spectrum is given by $E_{n_{eff}} = n_{eff}\ E_1$. The semi-classical coupled channels model [4b] is used to generate this result. The widths of the intermediate states are just the "harmonic" ones $\Gamma_n = n\Gamma_1$. Also shown is the cross-section from Ref. [3] (solid line) and the simple undamped harmonic oscillator cross-section also calculated according to [4b] (small dashed line). The results shown are for the system ^{208}Pb+^{208}Pb. See text for details.